

# Theoretical prediction of creep flux in aeolian sand transport

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## Abstract

The creep motion in aeolian sand transport is studied using two typical granular flow models. We focus on the expression of creep flux. It is theoretically revealed that creep fraction (the contribution of creep to the overall sand flux) changes with wind velocity and grain size. © 2004 Elsevier B.V. All rights reserved.

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## 1. Introduction

The reliable prediction of sand transport flux plays a fundamental role in the studies of aeolian geomorphology and wind erosion control. The motions of wind-driven grains were classified into three distinct modes, namely, suspension, saltation and creep, in the pioneering work of Bagnold [1]. To date, our knowledge of the process of aeolian sand transport has been improved significantly [2–6]. The fourth motion mode, reptation, was introduced by Ungar and Haff [2]. Following Anderson et al. [4], their definitions are: suspension is the transport mode in which grains are lifted far away from the surface and are transported over long distances without contact with the sand bed; saltation is defined as the transport mode of grains capable of rebounding or of splashing up other grains; reptation is referred to the motion of splashing grains which can not rebound or eject other grains; the motion of grains whose displacement is not affected directly by wind forces is defined as creep. Thus, the total sand flux consists of four parts. Except in the case of a very dusty sand, the contribution of suspension to total sand flux can be neglected [7,8].

The importance of saltation is well known. Many grains move in saltation and their collisions with sand bed can result in grain emission (suspension), reptation and creep. Numerous experimental, numerical and theoretical works

(see, for instance, [8–12] and references therein) have been devoted to saltation. Therefore, we have adequate models to calculate saltation flux. Besides, some expressions of reptation flux have been generated while researching the dynamics of aeolian sand ripples and dunes [6,13,14]. Compared with above fruitful results of saltation, reptation, etc., the understanding of creep is very limited at present. Early experimental results [1,15] differ quite dramatically because the sand samplers used in these experiments to directly measure creep flux could trap some saltating and reptating grains at the same time. For example, the creep fraction, obtained by different researchers, varies from 6.5% to 50%. Anderson et al. [4] remarked that measurement of creep is a delicate matter that had not yet been attempted. Recently, Dong et al. [16,17] reported the systematic wind tunnel results of the sand flux profiles for different grain sizes at different wind velocities, then extrapolated these sand flux profiles to sand bed and gave available information about creep fraction. As far as we know, there is no theoretical model to quantitatively predict creep flux.

On the other hand, creep is undoubtedly a granular flow which is of great interest in physicists [18,19]. Several theories have been developed to describe surface flows. Two simple continuum models have been proposed, based on the hypothesis that a partial flowing granular material can be separated into two phases, static and rolling. One is BCRE model [20] and its developments [21]. The other is Saint-Venant model [22,23]. The final equations in both models have the same structure [24]. They can reproduce certain observed features of granular flows and have been

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applied to deal with the dynamics of aeolian sand ripples [14]. In addition, some more general and rigorous models [25–27] have also been established. Unlike BCRE or Saint-Venant model, these “complete” models can give detailed properties of granular flows. For instance, the model proposed by Rajagopal and Massoudi [26] has been applied to various problems such as flow in a vertical pipe [28], flow due to natural convection [29] and flow between rotating cylinders [30]. Although modelling granular flow is still a challenge, we expect that the existent theoretical works will be helpful in determining creep flux.

In this paper, creep motion is simplified firstly; then, creep flux is discussed using two typical granular flow models; finally, theoretical predictions are compared with the recent wind tunnel data [16,17].

**2. Basic definition**

For a fully developed aeolian sand flow, saltating, reptating and creeping grains come to a dynamic equilibrium state (see Fig. 1). We choose axes such that the wind direction is  $x$ , and the direction perpendicular to sand bed is  $y$ . The interface between the saltation–reptation layer and the creep layer is  $y=0$ . The net mass exchange across this interface is zero. The effect of wind within the creep layer can be neglected because the wind velocity near sand surface is very small, and creep motion belongs to dense granular flows. Once surface grains of creep layer are impacted by saltating or reptating grains, the obtained momentum will be transmitted downward and forward within the creep layer through grain contacts such as frictions and collisions. We need know the boundary condition at  $y=0$  before discussing creep motion. It is often assumed that the sum of the airborne shear stress and the grain-borne shear stress is a constant [5,8,12]. To keep an equilibrium state of saltation motion, the saltating grain-borne shear stress must yield

$$\tau|_{y=0} = \rho_a U_*^2 \tag{1}$$

where  $\rho_a$ ,  $U_*$  are air density and friction velocity(or shear velocity), respectively. In an equilibrium state, the momen-

tum of saltating grains is entirely transmitted to creeping grains through reptating grains and direct collisions with surface grains of creep layer. Therefore, Eq. (1) is just the boundary condition of creep motion.

In the continuum description of a granular media, the bulk density can be written as

$$\rho = \rho_s v \tag{2}$$

where  $\rho_s$  is the density of a single sand grain;  $v$  is volume fraction which often is a function of spatial coordinates.

As mentioned above, we only study the creep motion in equilibrium. The continuity equation is satisfied automatically. The only nonzero velocity component is

$$V_x = u(y) \tag{3}$$

Given  $\rho$  and  $V_x$ , the creep flux,  $Q_c$ , can be calculated through

$$Q_c = \int_{y=-\infty}^0 \rho V_x dy = \int_{y=-\infty}^0 \rho_s v u(y) dy \tag{4}$$

**3. Model I**

It seems that the simplest way to get the expression of  $Q_c$  is using the results of surface flow directly. In this section, we choose Saint-Venant model [22,23] which reduces to only one depth-average equation, the  $x$ -momentum balance equation,

$$\tau|_{y=0} = \tau|_{y=-\delta} \tag{5}$$

where  $\delta$  is the thickness of surface flow. The shear stress is the sum of a collision contribution and a friction contribution [31]. The distinction between collision and friction is the different duration of contact between grains [32]. Considering binary collisions, Bagnold [33] introduced a simple relation between collisional stress and shear rate. The friction term is assumed to be of Coulombic form. Following Khakhar et al. [23,34], the shear stress at  $y=-\delta$  can be written as

$$\tau|_{y=-\delta} = c\rho d \left(\frac{\partial u}{\partial y}\right)^2 + \mu_d \rho g \delta \tag{6}$$

where  $c \approx 1.5$ ,  $d$  is the grain size,  $g$  is the acceleration of gravity, and  $\mu_d$  is the coefficient of dynamic friction.

The velocity distributions in surface flows are complex in details [35,36]. For simplicity, a linear velocity profile is assumed in Saint-Venant model

$$u = \frac{2U}{\delta} (y + \delta) \tag{7}$$

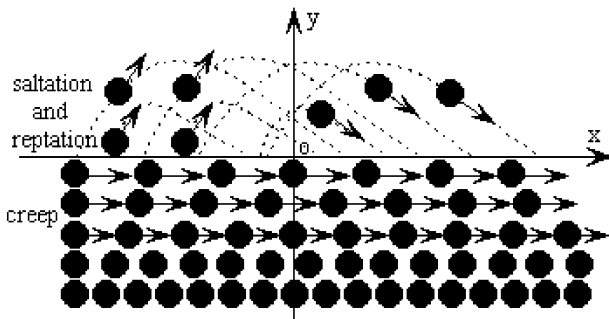


Fig. 1. An illustration of aeolian sand flow in the dynamic equilibrium state.

The restriction at  $y = -\delta$  is the Mohr–Coulomb criterion

$$\tau|_{y=-\delta} = \mu_m \rho g \delta \quad (8)$$

where  $\mu_m$  is the coefficient of static friction.

The constant volume fraction is supposed to be

$$v = \bar{v} \quad (9)$$

Combining Eqs. (1), (2) and (4)–(9), we obtain one part of creep flux due to surface flow

$$Q_{c1} = \int_{-\delta}^0 \rho_s \bar{v} u dy = AU_*^4 \quad (10)$$

$$\text{where } A = \frac{\rho_a^2}{2\rho_s \bar{v} g \mu_m^2} \sqrt{\frac{\mu_m - \mu_d}{cdg}}.$$

The grains at  $y < -\delta$  move much slowly than surface grains. The velocity distribution [37] is approximately

$$V = V|_{y=-\delta} \exp\left(\frac{y + \delta}{h_c}\right) \quad (11)$$

where  $h_c \propto d$ . The velocity  $V|_{y=-\delta}$  is so small that we ignore its influence upon the creep flux of surface flow, but it is a dominating term in Eq. (11). We must know some information about  $V|_{y=-\delta}$ . As done in many studies of aeolian transport flux, shear velocity is also selected as the primary parameter in this paper. After some dimensional arguments, it is found that a possible form is  $V|_{y=-\delta} \propto U_*$ . Therefore, another part of creep flux is

$$Q_{c2} = \int_{-\infty}^{-\delta} \rho V dy = B\rho_s d \bar{v} U_* \quad (12)$$

Finally, we get the total creep flux

$$Q_I = Q_{c1} + Q_{c2} = AU_*^4 + B\rho_s d \bar{v} U_* \quad (13)$$

#### 4. Model II

As a powerful tool, Navier–Stokes type constitutive equations give vivid descriptions of granular flows [27,38]. Neglecting the effects of bulk viscosity and volume fraction gradient in the original work of Rajagopal and Massoudi [26], such a constitutive equation is offered

$$T_{ij} = C_0 v \delta_{ij} + \frac{C_1 v(1+v)}{2} \left( \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) \quad (14)$$

where  $T_{ij}$  are components of Cauchy stress;  $C_0$  and  $C_1$  are constants; and, if  $i=j$ , then  $\delta_{ij}=1$ , otherwise  $\delta_{ij}=0$ . In Eq.

(14), the first term plays the role of pressure in an ordinary fluid. The second term corresponds to the viscous stress in Navier–Stokes equation. Furthermore,  $v > 0$  at all times. Thus, we conclude that  $C_0 < 0$  and  $C_1 > 0$ . In the particular case of creep motion discussed here, the shear stress and the normal stress can be written as

$$\tau = \frac{C_1 v(1+v)}{2} \frac{du}{dy} \quad (15)$$

$$N_y = C_0 v$$

Consider the balance of momentum, we have the following governing equations

$$C_0 \frac{dv}{dy} = \rho_s g v \quad (16)$$

$$\frac{d}{dy} \left[ v(1+v) \frac{du}{dy} \right] = 0 \quad (17)$$

The solution of Eq. (16) is

$$v = v_0 \exp\left(\frac{\rho_s g}{C_0} y\right) \quad (18)$$

which implies that  $v \rightarrow +\infty$  if  $y \rightarrow -\infty$  because  $C_0 < 0$ . This is unreasonable. In fact, there is a maximum possible volume fraction,  $v_{\max}$ , for every granular material. For spherical grains,  $v_{\max} = \frac{\pi}{3\sqrt{2}}$  [33]. From Eq. (18), we know that  $v = v_{\max}$  when  $y = y_1 = \frac{C_0}{\rho_s g} \ln\left(\frac{v_{\max}}{v_0}\right)$ . Let us suppose that once the volume fraction reaches the maximum value, sand grains will not flow.

$$u|_{y=y_1} = 0 \quad (19)$$

Substituting Eq. (18) into Eq. (17) and then integrating under the boundary conditions given by Eqs. (1) and (19), the velocity profile within the creep layer can be expressed analytically. The final form of creep flux is

$$Q_{II} = \int_{y_1}^0 \rho_s v u dy = CU_*^2 \quad (20)$$

$$\text{where } C = \frac{2\rho_a C_0^2}{\rho_s g^2 C_1} \left[ (1+v_0) \ln \frac{(1+v_0)v_{\max}}{(1+v_{\max})v_0} - \frac{v_{\max} - v_0}{v_{\max}} \right].$$

#### 5. Comparison between theory and experiment

Because the precise measurement of creep flux is lacking, the direct comparison of creep flux between theory and experiment is difficult. Here, we argue about an important

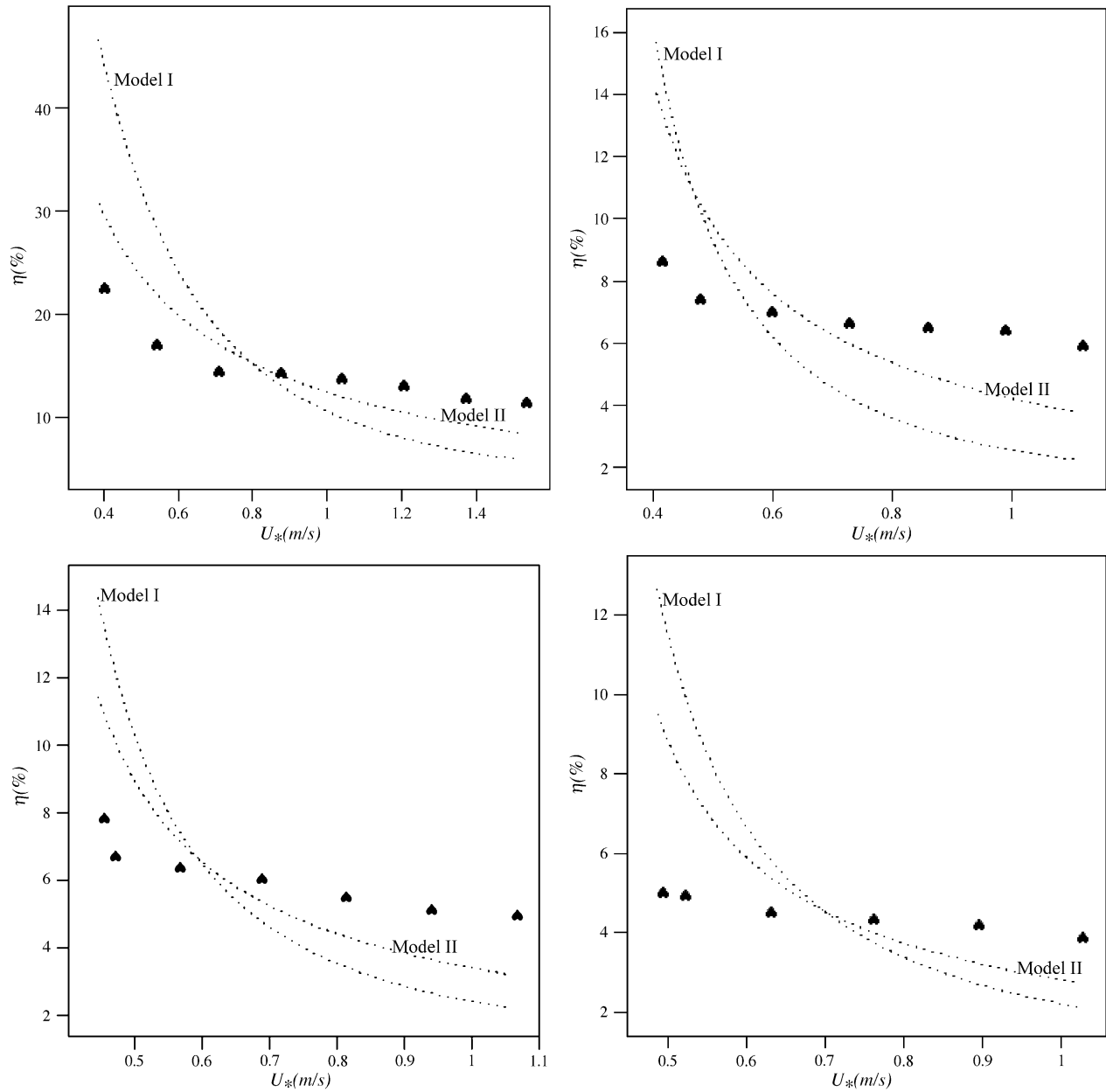


Fig. 2. Comparison between theoretical predictions and wind tunnel data [16]. The other parameters in calculations are  $\alpha=0.8$ ,  $\bar{v}=1/2 (v_0+v_{\max})$ ,  $\rho_s=2.65 \times 10^3 \text{ kg/m}^3$ ,  $\rho_a=1.225 \text{ kg/m}^3$ ,  $g=9.8 \text{ m/s}^2$ ,  $\mu_m=0.5$ ,  $\mu_d=0.4$ ,  $C_0=-0.232 \times 10^3 \text{ Pa}$ ,  $C_1=3.262 \times 10^3 \text{ kg/ms}$ . The symbols of  $\blacktriangle$  denote experimental data.

nondimensional parameter, creep fraction, which is defined as

$$\eta = \frac{Q_c}{Q_c + Q_{\text{sal}} + Q_{\text{rep}}} \quad (21)$$

where creep flux,  $Q_c$ , has been given by Eq. (13) or Eq. (20). The saltation flux  $Q_{\text{sal}}$  can be calculated easily. Many recommended saltation models are probably equally effective [5]. We select a straightforward modification of Bagdold model [1].

$$Q_{\text{sal}} = c \sqrt{\frac{d}{D} \frac{\rho_a}{g}} U_* (U_*^2 - U_{*t}^2) \quad (22)$$

where  $U_*$  is friction velocity; threshold friction velocity is  $U_{*t} = 0.1 \sqrt{\frac{\rho_s}{\rho_a} g d}$ ,  $D=0.25 \text{ mm}$ . Eq. (22) is remarkably close to Owen model [12]. Following Andreotti et al. [6], the reptation flux  $Q_{\text{rep}}$  is proportional to the saltation flux.

$$Q_{\text{rep}} = \alpha \frac{\sqrt{gd}}{U_*} Q_{\text{sal}} \quad (23)$$

where  $\alpha$  is a constant.

It is generally accepted that the friction velocity is a very important primary parameter when dealing with the mass flux of aeolian transport. This variable can be determined from the slopes of airflow velocity profiles [10]. However,

the information of wind velocity provided by Dong et al. [16] is inadequate for estimating the friction velocity. From their further treatment of wind tunnel data [17], we find

$$f_1(d) \left(1 - \frac{W_t}{W}\right)^2 W^3 = f_2(d) \left(1 - \frac{U_{*t}}{U_*}\right)^{0.25} U_*^3 \quad (24)$$

where  $W$  is the wind velocity at the centerline height of wind tunnel, and  $W_t$  is the threshold velocities.  $f_1(d)$  and  $f_2(d)$  are proportionality coefficients. They can be expressed by

$$f_1(d) = \frac{1}{475.24 + 93.62 \frac{d}{D}} \quad (25)$$

$$f_2(d) = 1.49 + 5.00 \exp\left(-\frac{1}{2} \left(\frac{\ln\left(\frac{d}{1.53D}\right)}{0.56}\right)^2\right)$$

The experimental results of creep fraction are related with the friction velocity through Eq. (24).

Fig. 2 presents the comparisons between theoretical predictions and wind tunnel data [16]. Both models indicate that if friction velocity is small, the creep flux can not be neglected when calculating the total sand flux; as friction velocity increases, the creep fraction decreases rapidly. This is consistent with the fact that aeolian sand ripples will disappear if the wind velocity is large enough. It is shown that the predictions of Model II are more accurate than that of Model I. The most important reason is that it is very difficult to determine the thickness of flowing layer in Model I. Describing the transition between rapid flow and dense slow flow is an open question [18]. While, there is no phase behavior explicitly in Model II.

Now, let us give some explanations of the differences between theoretical predictions and experimental results. First, Dong et al. [16] did not really investigate creep flux. The creep fractions were obtained through extrapolating sand flux profiles to sand bed. It is necessary to carefully estimate errors occurring in this measurement method. Second, both Eq. (13) and Eq. (20) are derived under the assumption that sand flux is everywhere saturated. But, there is no clear criterion to judge whether or not an aeolian sand flow is in equilibrium in wind tunnel experiments. Third, several parameters, including volume fraction, coefficients of friction and so on, still need be measured specially for creeping grains. An estimate of  $C_0$  and  $C_1$  is given in the Appendix A. Therefore, Fig. 2 is not so perfect as we expect.

## 6. Summary

In this paper, the creep motion in aeolian sand transport is studied using granular flow theories. Two formulae for creep flux are presented. One comes from Saint-Venant model directly. The other is derived from continuum mechanics model with Navier–Stokes type constitutive equations. A rough comparison between theoretical predictions and wind tunnel measurements is given. The results have

shown that the creep fraction is not a constant. It decreases rapidly with the increasing of friction velocity. The present theoretical work also reveals that the effect of creep can not be neglected in the calculation of total sand flux if wind velocity is not large enough.

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## Appendix A. Material parameters in model II

Many techniques have been developed to capture flow properties of granular material [39]. To measure material parameters in their model, Rajagopal et al built an orthogonal rheometer [40]. The test material in a cup is placed between two parallel plates which are rotating at the same angular speed and in the same direction, about non-coincident axes (see Fig. A1).

If the flow is slow, it can be assumed that the velocity field and the volume fraction have the following forms [40]

$$V_x = -\Omega[y - g(z)]$$

$$V_y = \Omega[x - f(z)]$$

$$V_z = 0$$

$$v = \text{const} \quad (A1)$$

Neglecting the term of order  $\Omega^2$ , the momentum balance equation in  $x$  direction reduces to

$$\frac{d^2 g}{dz^2} = 0 \quad (A2)$$

The behavior of granular materials at solid boundary is far from being well understood [36,40]. Note the system is symmetric about  $Oy$  and the flow velocity is small, we hope

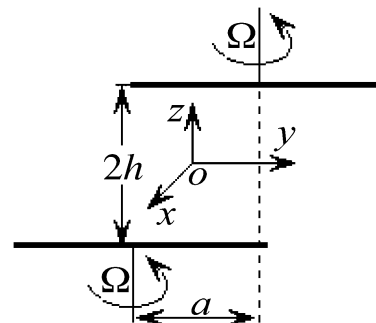


Fig. A1. Schematic diagram of the orthogonal rheometer [40].



that the granular material adheres to the boundary plates in  $x$  direction. This leads to

$$g(h) = -g(-h) = \frac{a}{2} \quad (\text{A3})$$

Thus, the solution of Eq. (A2) is

$$g = \frac{az}{2h} \quad (\text{A4})$$

Combining Eqs. (14), (A1) and (A4), the tractions on the plate are

$$t_x(\pm h) = \mp C_1 \frac{av(1+\nu)\Omega}{4h}$$

$$t_z(\pm h) = \mp C_0 \nu$$

The average measurements of the  $x$  and  $z$  component of the forces on the plates are

$$f_x = 5.75 \times 0.4536 \times 9.8N$$

$$f_z = 5.07 \times 0.4536 \times 9.8N$$

In the experiments [40], the cup height, the distance between two axes, the diameter of plates, the volume fraction and the angular speed are  $2h=0.5 \times 2.54 \times 10^{-2}$  m,  $a=0.62 \times 2.54 \times 10^{-2}$  m,  $d_p=4.5 \times 2.54 \times 10^{-2}$  m,  $\nu=0.68$  and  $\Omega=10 \times 2\pi/60$  rad/s, respectively.

Finally, two material parameters  $C_1$  and  $C_0$  in the constitutive equation (14) are obtained

$$C_1 = \frac{16f_x h}{av(1+\nu)\Omega\pi d_p^2} = 3.262 \times 10^3 \text{ kg/ms}$$

$$C_0 = -\frac{4f_z}{\nu\pi d_p^2} = -3.232 \times 10^3 p_a$$

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