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The mathematics teacher in shadow education: a new area of focus in teacher education

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Abstract
This paper presents a discussion on a new focus area in mathematics teacher education, namely the mathematics teacher in shadow education (SE mathematics teacher). It addresses three issues pertaining to SE mathematics teachers: special knowledge and expertise they need to possess, teacher qualification and selection and SE mathematics teacher professional development corresponding, respectively, to necessary specialized expertise. Some perspectives for future teacher education research are also addressed.

Keywords Mathematics teacher in shadow education · Regular school mathematics teacher · Teacher qualification · Teacher expertise · Teacher professional development

Introduction

Private supplementary tutoring is known as shadow education and has become an international phenomenon in recent decades (Bray, 2009; Cole, 2016; He et al., 2021; Liu, 2012). Traditionally, it was more popular in East Asian regions such as China, Korea, Japan, and Singapore, mainly because of exam culture caused by tense competition in high-stakes examinations (Wang & Guo, 2017). In these regions, shadow education is a huge commercial market and family cost (Bray, 2014; Byun et al., 2018; Yung & Chiu, 2020; Zhang et al., 2020). The popularization of shadow education expanded the world over in the past few decades (Bray, 2014; Byun et al., 2018; Guill et al., 2019). Mathematics is usually a gatekeeper subject in high-stakes examinations (Burkhardt, 2007; Wang & Guo., 2017) and therefore receives more attention in the shadow education market (Zhang et al., 2021).

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Typically, shadow education means paid private supplementary tutoring or coaching aimed at providing additional help to students outside regular school to prepare for a variety of academic examinations (Bray, 2009); and teaching is the main activity. Teaching in shadow education might have specific principles different to regular school teaching since teaching in shadow education might focus on content taught in regular school and hopes to enhance the students' academic achievement in regular school with relatively short, temporary teaching. Since mathematics teaching and teacher education are emphasized in mathematics education research, we would also argue in a same way: mathematics teaching and teachers in shadow education are also worth considering. In addition, mathematics teachers in shadow education should logically be a specific category of mathematics teachers, although, in practice, some of them used to be regular school teachers or received teacher education for regular school teachers.

Typical studies (Pepin et al., 2016) on (mathematics) teacher education aim at improving teacher quality, as it decides teaching quality, which on the one hand is linked with students' learning, and on the other hand, benefits the construction of the education system. Studies have focused on various issues ranging from recruitment, preparation, and induction to ongoing professional development and teacher assessments (Darling-Hammond, 2017; Darling-Hammond et al., 2002; Leman, 2001; Sowder, 2008), covering both preservice and in-service stages (Even & Ball, 2009). Issues in the teacher education system were widely and deeply discussed around teacher training and teacher knowledge, skill, competency, expertise, beliefs, learning, etc. (Ponte & Chapman, 2008; Liljedahl et al., 2009; Tatto & Lerman, 2009). The diverse points of discussion mentioned above can be categorized into three issues: what is a qualified teacher (expertise), how to be a teacher (qualification), and how to become a qualified teacher (professional development). This commentary aims to illustrate the importance of exploring mathematics shadow teachers, especially regarding the three key issues in mathematics teacher education.

With the rapid development of shadow education globally, the market needs more employees to join the tutoring trades, which brings a growing population of teachers working in shadow education, especially for mathematics. We assumed that mathematics teachers who work in shadow education tend to become a new specialized category of mathematics teachers since they also teach mathematics, but their functions and tasks are different from regular mathematics schoolteachers. Considering the meaning of "shadow" and its nature of mimicking the regular school teaching, some questions arise: Could we reconsider shadow education with the research approaches and ideas that we are adapting in the regular school system? Are these mathematics teachers as qualified as the regular schoolteachers are? Is the expertise necessary and sufficient to handle shadow education the same as that in regular schools, by just moving the regular school classroom outside the school environment? Or is there a distinct need for new knowledge/expertise for teachers to handle the jobs in shadow education? Do professional mathematics teachers need specific expertise to engage in shadow education? Are the teacher professional development programs that work in regular schools suitable for shadow education?

The expanded research on shadow education mainly looks at a macro level of economics or policy (such as Bray, 2009), and seldom goes into the pedagogy or the preparation of teachers for shadow education. Its impact on the mainstream education system are partly due to the increased importance of research related to teachers training and education in shadow education.

In this study, we refer to these teachers collectively as SE (Shadow Education) mathematics teachers. The teachers who are considered typical SE teachers have the following specific characteristics: assistance is provided outside regular schools; it focuses mainly on

academic subjects that have already been covered in school; and it supports preparation for high-stakes tests and requires private financial outlays that bring them extra gains, among other things (Baker et al., 2001; Bray, 1999, 2003; Kuan, 2011). These teachers are also known as tutors (who work individually as freelancers) or cram school teachers (who work for cram schools/agencies), and it is clear that the cram school teachers could receive support for professional development and monitoring while the tutors might not, which makes it necessary to differentiate the two categories of teachers in shadow education. It should be noted that few studies have examined this kind of mathematics teacher in depth. In this study, we do not include schoolteachers who provide additional instruction to students after regular school time for free or paid for by the government as tutoring programs in the scope of our discussion on SE mathematics teachers. These programs tend to be a part or extension of regular schooling and the teachers need more the expertise of regular school teachers than typical SE teachers. Although the tutoring program might also benefit from a study of typical SE teachers, we need first to concentrate on the studies of typical SE teachers at the initial stage of this new domain of mathematics teacher education.

The three sections in this paper discuss the need for new issues in the field of mathematics teacher education, namely, the specialized knowledge and expertise that SE mathematics teachers need to possess; their qualifications and selection process; and their professional development. Following this, the paper proposes directions for future research.

The specialized expertise needed by SE mathematics teachers

Expertise is often defined along with the notion of "expert" when referring to the characteristics, skill, and knowledge that distinguish experts from the masses (Ericsson, 2006). Many studies related to expertise concern the knowledge aspect (Ball et al., 2008; Hill & Ball, 2004) and resulted in different categories of knowledge, such as the famous pedagogical content knowledge (PCK) of Shulman and his colleagues (Ponte & Chapman, 2008; Shulman, 1987; Wilson et al., 1987).

In shadow education, SE mathematics teachers also need some level of expertise to fulfill their requirements in the teaching of shadow education. The mathematics teaching of shadow education is also a professional activity, though it generally focuses on supplementing regular school teaching, which might involve a specific principle. Unfortunately, due to the lack of research focus on teacher education in shadow education, we did not find any specific studies on the principles and expertise that SE mathematics teachers should be equipped with. Some scholars might even think that there is no specific expertise needed for SE mathematics teachers.

It should be noted that there is not a chasm between expertise for SE mathematics teachers and regular school teachers, especially with the fact that some SE mathematics teachers used to be regular school teachers. However, the two kinds of expertise tend to have different focuses in practice. Expertise for SE mathematics teachers might generally be the same as what is expected for regular school teaching, such as mathematical content knowledge and PCK, but the extent and preference could be diverse. The concrete focus of teaching methods and expertise is different, along with the service prepared for their consumers (students or parents) and the key performance indicators (KPIs) (required by the shadow education enterprises. SE mathematics teachers, as the name "shadow" implies, often cope with the problems that students have met or could meet in their regular school learning. By helping them review or preview the school

lessons, SE teachers aim to provide their consumers with specific support to conquer learning difficulties they have met or could meet, close the learning gap. Their work often needs to start with diagnosing or assessing how well their students have learned in the regular school, as students usually come with problems in relation to what they were taught by their mathematics teachers at school. They analyze the students' learning difficulties, learning style, school performance, level of understanding of mathematics, problems encountered when using different learning methods, and even non-cognitive problems in learning, such as their learning interest, motivation, and meta-cognition of mathematics (Alexander et al., 2003; Wang et al., 2016).The next step is to provide supplemental teaching, such as training of exam skills (Yung, 2020), training of fluency of mathematical procedure application, and so on. In contrast, regular school teaching tends to pay more attention to the teaching of new mathematics content with a focus on powerful mathematics ideas, such as discussing why a particular mathematical statement is true or where a mathematical rule comes from, using discovery learning, inquiry teaching, or project-based learning, etc. Owing to the different missions and nature of shadow education, SE mathematics teachers, and schoolteachers, it is worth studying their specialized expertise. The term "specialized" here does not refer to expertise that is only necessary for SE mathematics teachers but also includes expertise that can help them work better than regular teachers in some ways, such as in carrying out diagnostic assessments of student learning and implementing one-to-one instruction strategies, since they generally need to teach students the content that has been learned in formal school.

In the practice of shadow education, the consumers (students or parents) have a clear need: improved exam scores (Davies, 2004). These market requirements cause shadow education to differ from the school system: SE mathematics teachers might not need to have educational missions; only the responsibility to design products (e.g. courses or exercises) related to exams or mathematics knowledge (Yung, 2020). Shadow education tend to care more about whether SE mathematics teachers can satisfy their consumers' demands and attract more consumers to buy their courses, which might impact the teaching of shadow education.

Additionally, the mission of shadow education involving enhancing students' academic scores should be reworked. Are improved academic scores the only criterion for evaluating SE mathematics teachers? If not, what other criteria could be considered? The issue of the self-perceptions and beliefs of SE teachers remains to be considered: should SE mathematics teachers make the students work harder and do better at school so that they do not need extra tutoring in the future (although this is not the expectation of cram schools)? The mission should not be limited to preparing for examinations, although consumers treat this as a major requirement. "Tutoring for not tutoring anymore" could be considered a kind of teacher belief for SE mathematics teachers.

Based on the data from Hong Kong English teacher, Yung (2020) indicated that students tended to perceive SE teachers as better than schoolteachers in increasing their learning motivation, and their suggestions and instructions were seen as more acceptable among the students, which provided a direction for future studies on SE mathematics teachers by comparing the teaching of SE mathematics teachers with that of formal schoolteachers from the student perceptions and illustrated that characteristic of SE mathematics teachers. Other factors could include teaching style (humor and funny quirks, and sometimes even exaggerated explanations) and young and trendy appearance. In some public schools, teachers are asked to hold to a dress code, such as knee-length skirts, and perform like teachers, which does not strictly suit SE mathematics teachers, who focus on attracting and

impressing students. However, these could make students feel fresh and relaxed, especially when their schoolteachers tended to be "conservative," "old-school," and "severe," especially in Eastern-Asian cultures.

More studies are necessary to discuss teacher expertise in detail, especially when the mission of shadow education shifts from skills training to actually providing support for regular school learning with a broader mission, such as helping students acquire powerful mathematics ideas.

The qualifications and selection of SE mathematics teachers

The subject of qualifications raises the question of who can be an SE teacher, with regards to entry, quality, and selection? With the rise in the number of SE mathematics teachers, qualifications with clear standards become an urgent problem, particularly in ensuring teacher quality and/or appropriate teacher qualifications to serve as evidence in the selection of these teachers.

It is not a rare phenomenon in practice, that SE teachers have neither received formal teacher education training nor majored in mathematics or science. They may be engineers, university students (whether they major in teaching or not), or even high school students or graduates. High school students or graduates could teach primary and middle school students, as long as they have some expertise with the content (Liu & Bray, 2020; Ömeroğulları et al., 2020).

The various backgrounds of SE mathematics teachers result in uneven quality, as well as causing issues with the selection and qualification of teachers. The background of SE mathematics teachers was also discussed with regard to the teaching efficiency of shadow education. Some studies have applied the background of an SE mathematics teacher as a superficial standard for qualification, such as whether they are secondary school or university students, or trained teachers or working for a tutoring center (Ömeroğulları et al., 2020; Wang & Guo, 2017). Furthermore, a German study found that compared to university students, regular schoolteachers and cram school teachers even have a significantly negative effect on students' mathematics grades and test scores (Ömeroğulları et al., 2020). This counterintuitive finding deserves further study, since it contrasts with the assumption of many consumers (parents), who generally pay higher prices to hire regular or cram school teachers rather than university students to tutor their children, Ömeroğulları et al., (2020) suggest that future studies should assess the specific qualification of tutors in tutoring centers in more detail. The indicator of the qualification of regular schoolteacher centers on teacher experience and educational background (Wang et al., 2018), as well as the title system (according to a set standard) and promotion logic (although this is quite different from country to country).

The process of selecting an SE mathematics teacher is quite different from that of a regular schoolteacher. The former is decided by the cram school (employers) and consumer (families) or by the consumer alone. Consumers might not have related expertise, and the regular school system usually has a mature structure and is supported by a wide range of educational studies. Few studies have examined the qualifications and requirements of SE teachers (e.g. Ömeroğulları et al., 2020; Zhang et al., 2021). We did not find any research on SE teacher recruitment and selection, which is worthy of attention.

A regular school can select and hire teachers based on a list of criteria and carry out a professional evaluation for teachers accordingly. In practice, cram schools and the families

(consumers) might emphasize the educational background of SE mathematics teachers, especially when they are studying in or have graduated from well-acknowledged universities or have experience working in famous regular schools with excellent track records. Based on the data from Hong Kong, Yung (2020) found that if the cram schools apply more resources to advertising their SE teachers' qualifications, it often results in greater trust in their shadow teachers by the consumers. However, such advertisements cannot always guarantee good efficiency. Families might not be able to judge whether an SE mathematics teacher could help (even some cram school leaders might not have the required knowledge), especially when their children have severe and implicit learning difficulties and they are looking for specific help that demands high-quality SE mathematics teaching.

In practice, the logic of teacher evaluation in shadow education is different from that for regular schoolteachers (Ömeroğulları et al., 2020), partly because of the nature of their jobs: They may need to take charge of their KPIs, such as how many consumers buy their courses and how many of them decide to continue after the class schedule. SE teachers might have the pressure of losing their jobs or reducing their income. Their KPIs are designed more from an enterprise's perspective, such as how to satisfy their customers on a priority basis, but not necessarily to meet the aims of education (Yung, 2020).

The presence of non-uniform qualifications for SE mathematics teachers does not mean that no specific qualifications are necessary to become an SE mathematics teacher. It is essential to examine the kind of expertise that is necessary to become a high-quality SE mathematics teacher in order to standardize qualifications. Future research should explore how the qualifications and selection processes for SE mathematics teachers should be developed according to an updated mission of shadow education, as well as the specialized expertise that SE mathematics teachers need.

Moreover, after the discussion on the qualification and expertise, we will consider how SE mathematics teachers should be trained to gain professional development and become high-quality teachers, especially for those who did not take part in teacher education programs.

The professional development of shadow mathematics teachers

As discussed above, the background of SE mathematics teachers are varied, and there are several specialized areas of expertise among SE mathematics teachers that need to be developed even among regular schoolteachers when they begin to teach shadow education. In-service professional development activities form an essential part of the work of SE mathematics teachers; however, the current typical regular schoolteacher professional development program (Franke et al., 2001; Sowder, 2008) may not be suitable for meeting this need.

In practice, for example, some Chinese cram schools are also trying to train their SE mathematics teachers and support their professional development journeys in an economical and direct manner, especially when the demand for SE mathematics teachers is huge and employees with different backgrounds teach shadow education. In practice, the staff training in cram schools operate based on their teachers' professional and/or educational background. They are generally divided into professional and non-professional groups, and professional development programs are designed accordingly. By default, it is assumed that regular schoolteachers seeking new roles as SE mathematics teachers have the potential to do well in the field after undergoing complementary training to make the shift. For

teachers with non-professional background, it is generally reported that a (short) mathematics teacher training program would be provided, with a focus on the fundamental knowledge of mathematics instruction. It seems challenging for the professional development system at the cram schools for teachers with non-professional background, since in teacher education at universities or colleges, it generally takes one or more years' courses, including both theoretical and practical parts, which is diverse from country to country (Ponte & Chapman, 2008). We also learned that some SE mathematics teachers had spontaneously formed informal learning communities (Stein et al., 1998), which might be borrowed from regular school systems such as the Teaching Research Group in the Chinese school system (Yang, 2009), through which schoolteachers learned and worked together, shared teaching resources, exchanged notes on their challenges and solutions, and shared problems they encountered at work.

The existing practice offers a new direction for research on professional development in mathematics teacher education. This can not only help enhance the quality of professional development of SE mathematics teachers, but also has implications for the enhancement of the quality and efficacy of regular school professional development programs.

Implications for future research

Through a short discussion on the issues encountered by SE mathematics teachers, this commentary calls for a new focus on mathematics teacher education. An SE mathematics teacher is worth paying attention to in research, especially because their expertise differs from that of regular schoolteachers. Such efforts can not only contribute toward theoretical research on teacher education but can also help design effective teacher training programs and forward-facing educational policies to benefit teachers further.

The specialized expertise of SE mathematics teachers needs more case studies that can help the educational community understand how SE mathematics teachers can help their students obtain better learning outcomes (more than mere scores in mathematics examinations). The typical methodology that has been relied on to study regular schoolteachers, such as presage/process–product research (Dunkin & Biddle, 1974), and novice-experts research (Fuller & Unwin, 2010), can be adapted carefully.

The qualifications and selection processes in place for SE mathematics teachers, especially those in part-time arrangements and those with non-professional background, should be considered and addressed in the educational policy in order to better regulate the shadow education market.

Finally, it is also worth studying mature and successful professional development programs relied on by SE mathematics teachers, either organized by their cram schools or spontaneously pursued on their own terms. Paying attention to SE mathematics teachers, especially the logic of how they were trained in the enterprise system, might result in a win–win situation: both the cram schools and regular schools could reflect on their own experiences and borrow/share some experiences from each other for better training their employees.

Declarations

Conflict of Interest The authors declare that there are no competing interests.

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